Learning the Parameters of Deep Probabilistic Logic Programs

Arnaud Nguembang Fadja   Fabrizio Riguzzi   Evelina Lamma

Dipartimento di Ingegneria – University of Ferrara

Dipartimento di Matematica e Informatica – University of Ferrara
[arnaud.nguembafadja,fabrizio.riguzzi,evelina.lamma]@unife.it

University of Ferrara

Introduction

- Probabilistic logic programming is a powerful tool for reasoning with uncertain relational models
- Learning probabilistic logic programs is expensive due to the high cost of inference.
- We consider a restriction of the language of Logic Programs with Annotated Disjunctions called hierarchical PLP in which clauses and predicates are hierarchically organized.
- Inference is then much cheaper.

University of Ferrara

PLP under the Distribution Semantics

- A PLP language under the distribution semantics with a general syntax is Logic Programs with Annotated Disjunctions (LPADs)
- Heads of clauses are disjunctions in which each atom is annotated with a probability.
- LPAD T with n clauses: $T = \{C_1, \ldots, C_n\}$.
- Each clause C_i takes the form:

$$h_{i1} : \pi_{i1}; \ldots ; h_{iv_i} : \pi_{iv_i} :- \ b_{i1}, \ldots , b_{iu_i}$$

- Each grounding $C_i\theta_j$ of a clause C_i corresponds to a random variable X_{ij} with values $\{1, \ldots , v_i\}$
- The random variables X_{ij} are independent of each other.

University of Ferrara

Example

- UW-CSE domain:

$$
\begin{array}{l}
advisedby(A, B) : 0.3 :- \\
\quad student(A), professor(B), project(C, A), project(C, B). \\
advisedby(A, B) : 0.6 :- \\
\quad student(A), professor(B), ta(C, A), taughtby(C, B).
\end{array}
$$

Hierarchical PLP

- We want to compute the probability of atoms for a predicate r: $r(\mathbf{t})$, where $\mathbf{t}$ is a vector of constants.
- $r(\mathbf{t})$ can be an example in a learning problem and r a target predicate.
- A specific form of an LPADs defining r in terms of the input predicates.
- The program defined r using a number of input and hidden predicates disjoint from input and target predicates.
- Each rule in the program has a single head atom annotated with a probability.
- The program is hierarchically defined so that it can be divided into layers.
- Each layer contains a set of hidden predicates that are defined in terms of predicates of the layer immediately below or in terms of input predicates.

Hierarchical PLP

- Generic clause C:

$$C = p(\mathbf{X}) : \pi :- \phi(\mathbf{X}, \mathbf{Y}), b_1(\mathbf{X}, \mathbf{Y}), \ldots, b_m(\mathbf{X}, \mathbf{Y})$$

  where $\phi(\mathbf{X}, \mathbf{Y})$ is a conjunction of literals for the input predicates using variables $\mathbf{X}, \mathbf{Y}$.
- $b_i(\mathbf{X}, \mathbf{Y})$ for $i = 1, \ldots, m$ is a literal built on a hidden predicate.
- $\mathbf{Y}$ is a possibly empty vector of variables existentially quantified with scope the body.
- Literals for hidden predicates must use the whole set of variables $\mathbf{X}, \mathbf{Y}$.
- The predicate of each $b_i(\mathbf{X}, \mathbf{Y})$ does not appear elsewhere in the body of C or in the body of any other clause.

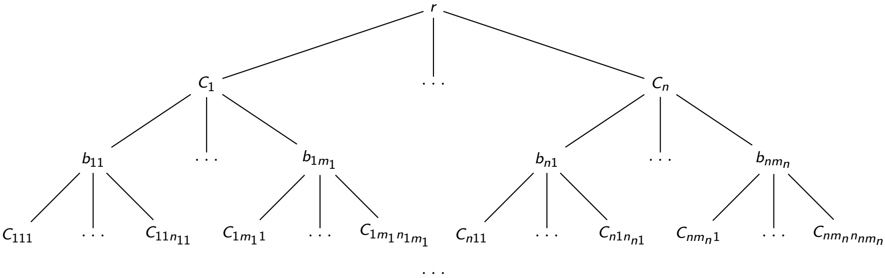
Hierarchical PLP

- A generic program defining r is thus:

$$
\begin{array}{rcl}
C_1 = r(\mathbf{X}) : \pi_1 & :- & \phi_1, b_{11}, \ldots, b_{1m_1} \\
 & \ldots & \\
C_n = r(\mathbf{X}) : \pi_n & :- & \phi_n, b_{n1}, \ldots, b_{nm_n} \\
C_{111} = r_{11}(\mathbf{X}) : \pi_{111} & :- & \phi_{111}, b_{1111}, \ldots, b_{111m_{111}} \\
 & \ldots & \\
C_{11n_{11}} = r_{11}(\mathbf{X}) : \pi_{11n_{11}} & :- & \phi_{11n_{11}}, b_{11n_{11}1}, \ldots, b_{11n_{11}m_{11n_{11}}} \\
 & \ldots & \\
C_{n11} = r_{n1}(\mathbf{X}) : \pi_{n11} & :- & \phi_{n11}, b_{n111}, \ldots, b_{n11m_{n11}} \\
 & \ldots & \\
C_{n1n_{n1}} = r_{n1}(\mathbf{X}) : \pi_{n1n_{n1}} & :- & \phi_{n1n_{n1}}, b_{n1n_{n1}1}, \ldots, b_{n1n_{n1}m_{n1n_{n1}}} \\
 & \ldots &
\end{array}
$$

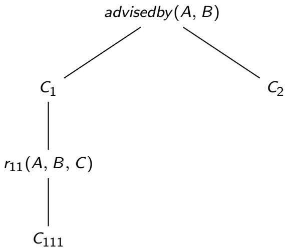
University of Ferrara

Program Tree



Example

$$
\begin{aligned}
C_1 \quad &= \quad advisedby(A, B) : 0.3 :- \\
&\qquad student(A), professor(B), project(C, A), project(C, B), \\
&\qquad r_{11}(A, B, C). \\
C_2 \quad &= \quad advisedby(A, B) : 0.6 :- \\
&\qquad student(A), professor(B), ta(C, A), taughtby(C, B). \\
C_{111} \quad &= \quad r_{11}(A, B, C) : 0.2 :- \\
&\qquad publication(D, A, C), publication(D, B, C).
\end{aligned}
$$



Inference

- Generate the grounding.
- Each ground probabilistic clause is associated with a random variable whose probability of being true is given by the parameter of the clause and that is independent of all the other clause random variables.
- Ground clause $C_{\mathbf{p}i} = a_{\mathbf{p}} : \pi_{\mathbf{p}i} :- \ b_{\mathbf{p}i1}, \ldots, b_{\mathbf{p}im_{\mathbf{p}}}$. where $\mathbf{p}$ is a path in the program tree
- $P(b_{\mathbf{p}i1}, \ldots, b_{\mathbf{p}im_{\mathbf{p}}}) = \prod_{i=k}^{m_{\mathbf{p}}} P(b_{\mathbf{p}ik})$ and $P(b_{\mathbf{p}ik}) = 1 - P(a_{\mathbf{p}ik})$ if $b_{\mathbf{p}ik} = \neg a_{\mathbf{p}ik}$.
- If a is a literal for an input predicate, then $P(a) = 1$ if a belongs to the example interpretation and $P(a) = 0$ otherwise.

Inference

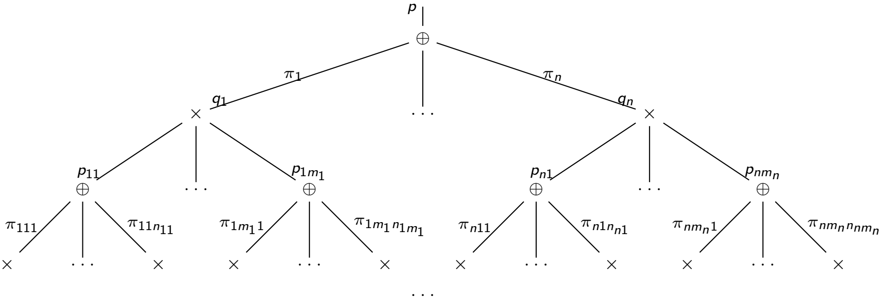
- Hidden predicates: to compute $P(a_{\mathbf{p}})$ we need to take into account the contribution of every ground clause for the predicate of $a_{\mathbf{p}}$.
- Suppose these clauses are $\{C_{\mathbf{p}1}, \ldots, C_{\mathbf{p}n}\}$.
- For one clause, $P(a_{\mathbf{p}}) = \pi_{\mathbf{p}1} \cdot P(body(C_{\mathbf{p}1}))$
- If we have two clauses,
  $P(a_{\mathbf{p}}) = 1 - (1 - \pi_{\mathbf{p}1} \cdot P(body(C_{\mathbf{p}1})) \cdot (1 - \pi_{\mathbf{p}2} \cdot P(body(C_{\mathbf{p}2})))$
- $p \oplus q \triangleq 1 - (1 - p) \cdot (1 - q)$.
- This operator is commutative and associative:

$$\bigoplus_i p_i = 1 - \prod_i (1 - p_i)$$

- The operators $\times$ and $\oplus$ are respectively the t-norm and t-conorm of the product fuzzy logic [Hajek 98]: product t-norm and probabilistic sum.

Inference

- If the probabilistic program is ground, the probability of the example atom can be computed with the arithmetic circuit:



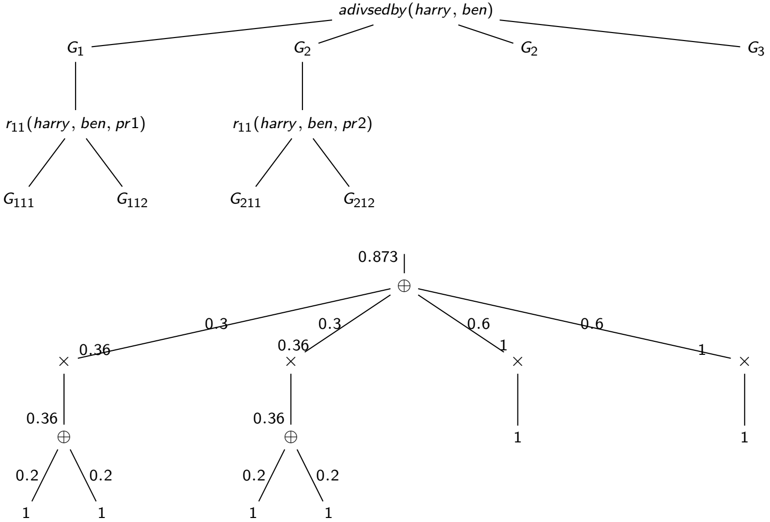
- The arithmetic circuit can be interpreted as a deep neural network where nodes have the activation functions $\times$ and $\oplus$

Example

$$
\begin{aligned}
G_1 &= advisedby(harry, ben) : 0.3 :- \\
&\quad student(harry), professor(ben), project(pr1, harry), \\
&\quad project(pr1, ben), r_{11}(harry, ben, pr1). \\
G_2 &= advisedby(harry, ben) : 0.3 :- \\
&\quad student(harry), professor(ben), project(pr2, harry), \\
&\quad project(pr2, ben), r_{11}(harry, ben, pr2). \\
G_3 &= advisedby(harry, ben) : 0.6 :- \\
&\quad student(harry), professor(ben), ta(c1, harry), taughtby(c1, ben). \\
G_4 &= advisedby(harry, ben) : 0.6 :- \\
&\quad student(harry), professor(ben), ta(c2, harry), taughtby(c2, ben). \\
G_{111} &= r_{11}(harry, ben, pr1) : 0.2 :- \\
&\quad publication(p1, harry, pr1), publication(p1, ben, pr1). \\
G_{112} &= r_{11}(harry, ben, pr1) : 0.2 :- \\
&\quad publication(p2, harry, pr1), publication(p2, ben, pr1). \\
G_{211} &= r_{11}(harry, ben, pr2) : 0.2 :- \\
&\quad publication(p3, harry, pr2), publication(p3, ben, pr2). \\
G_{212} &= r_{11}(harry, ben, pr2) : 0.2 :- \\
&\quad publication(p4, harry, pr2), publication(p4, ben, pr2).
\end{aligned}
$$

University of Ferrara

Example


adivsedby(harry, ben)
G1
G2
G2
G3
r11(harry, ben, pr1)
r11(harry, ben, pr2)
G111
G112
G211
G212
0.873
0.3
0.3
0.6
0.6
0.36
0.36
1
1
0.36
0.36
0.2
0.2
0.2
0.2
1
1
1
1
1
1

Example

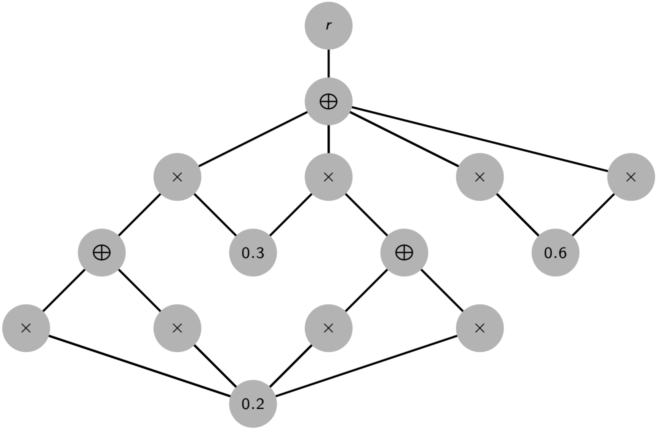


Figure: Arithmetic circuit.

.

Parameter Learning

- Given a Hierarchical PLP T with parameters Π, an interpretation I defining input predicates and a training set $E = \{e_1, \ldots, e_M, \textbf{not}\ e_{M+1}, \ldots, \textbf{not}\ e_N\}$ find the values of Π that maximize the log likelihood:

$$\arg\max_{\Pi} \sum_{i=1}^{M} \log P(e_i) + \sum_{i=M+1}^{N} \log(1 - P(e_i)) \quad (1)$$

  where $P(e_i)$ is the probability assigned to e_i by $T \cup I$.
- Maximizing the log likelihood can be equivalently seen as minimizing the sum of *cross entropy errors* err_i for all the examples

$$err_i = -y_i \log(p_i) - (1 - y_i) \log(1 - p_i) \quad (2)$$

  where $y_i = 1$ for positive example, $y_i = 0$ otherwise and p_i the probability that the atom is true.

Parameter Learning

- Partial derivative of the error with respect to each node $v(n)$:

$$\frac{\partial err}{\partial v(n)} = \begin{cases} -\frac{1}{v(r)} d(n) & \text{if } e \text{ is positive,} \\ \frac{1}{1-v(r)} d(n) & \text{if } e \text{ negative.} \end{cases}$$

where

$$d(n) = \begin{cases} d(p_n)\frac{v(p_n)}{v(n)} & \text{if n is a } \bigoplus \textit{node}, \\ d(p_n)\frac{1-v(p_n)}{1-v(n)} & \text{if n is a } \times \text{ node} \\ \sum_{p_n} d(p_n).v(p_n).(1-\Pi_i) & \text{if n is a leaf node } \Pi_i \\ -d(p_n) & p_n = not(n) \end{cases} \quad (3)$$

and $v(n)$, p_n are respectively the value and the parent of the node n.

University of Ferrara

Parameter Learning

- Build the ACs and initialize the parameters and the gradients.
- Perform the forward pass by computing the output of each node ($v(n)$) in the AC.
- Compute the gradient of the error w.r.t the output and back-propagate.
- Update the parameters using Adam optimizer.
- Until convergence or a certain condition is satisfied.

Conclusion and Future Work

- Conclusion
  - hierarchical PLP: a restriction of the language of LPADs that allows to perform inference quickly using a simple and cheap dynamic programming algorithm such as PITA(IND,IND).
  - Programs can be seen as arithmetic circuits/neural networks.
  - Parameters can be trained by gradient descent and back-propagation.
- Future work
  - Perform experiments.
  - Parameter learning by Expectation Maximization.
  - Perform also structure learning.

University of Ferrara

THANKS FOR LISTENING

AND

ANY QUESTIONS ?

University of Ferrara